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VISCOUS ATTENUATION OF THE FOURIER COMPONENTS OF A SHOCK WAVE F--ETC(U)
SEP 50 A B ARONS, G K WERTHEIM
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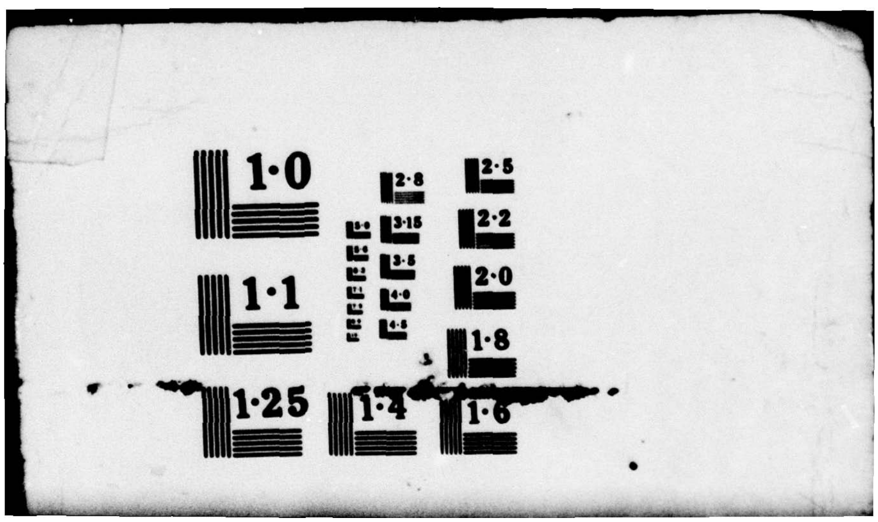
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Reference No. 50-32

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Viscous Attenuation of the Fourier Components of a Shock Wave from the Explosion of a One-Half Pound Charge in Sea Water,

By

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12 14 p.

Submitted to the Bureau of Ships
Under Contract NObsr-43270

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11 Sep[redacted]50

APPROVED FOR DISTRIBUTION _______________
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I INTRODUCTION

1.1 The methods and results of reference (1) are applied to the calculation of the relative energy distribution among the frequency components of a shock wave at various distances from the explosion of a one-half pound charge in sea water. The mathematical details are summarized in Appendix I. Qualitatively, the method used and approximations involved may be described as follows:

(a) At distances relatively close to the charge, the shock wave is represented as an exponentially decaying wave with a mathematically discontinuous front.

(b) As the shock wave travels outward, viscous effects cause the selective attenuation of higher frequency components in the manner described empirically by Liebermann (2). This produces a change in the shape of the shock wave, rounding off the front and decreasing the relative energy content in the higher frequencies.

(c) Owing to finite amplitude effects, the profile of the shock wave is continually spreading, resulting in a continually increasing time constant. Also the rounding of the front by viscous attenuation is partly opposed by the effect of finite amplitude in restoring higher frequency components and sharpening the front. These effects are treated by means of various approximations described in reference (1). The essence of the method lies in the calculation of an "equivalent viscous distance", x , which represents the distance that the shock wave would travel in the absence of finite amplitude effects in order to be subjected to the same effective attenuation of higher frequency components as it actually experiences in travelling the distance R under the influence of the superposed effects of viscosity and finite amplitude.

II FREQUENCY SPECTRUM OF THE ATTENUATED WAVE AT VARIOUS DISTANCES FROM A 1/2 LB. CHARGE

2.1 Figure 1 shows the energy distribution function (equation (17), Appendix I) plotted against frequency. Energy flux is given as db above 10^{-6} ergs/cm^2 for various distances from the explosion. The data are given in the frequency range for which the method of analysis is reasonably accurate. At frequencies lower than about 0.5 to 1 kc, equations (12) and (17) of Appendix I would be increasingly in error since it is obvious that the explosion wave is not one of infinite duration and that therefore the

low frequency spectrum as given by the exponential wave of equation (1) has no physical meaning. The actual distribution function must curve down and reach zero at zero frequency. At frequencies higher than 30 kc, the method of reference (1) introduces too severe an attenuation by neglecting the relaxation effect described by Liebermann(2).

2.2 Figure 2 shows the same data as Figure 1, but with the energy distribution function plotted against distance for various frequencies.

2.3 Tables of the numerical results on which Figures 1 and 2 are based are given in Appendix II.

APPENDIX I

Mathematical Development

1. The Unattenuated Shock Wave.

The initial, undistorted shock wave produced by the underwater explosion may be represented as an exponentially decaying pulse with a mathematical discontinuity at time: $t = o$

$$p(t) = \begin{cases} P_o \epsilon^{-\lambda t} & , t > 0 \\ 0 & , t < 0 \end{cases} \tag{1}$$

where λ is the reciprocal of the time constant θ and P_o is the peak pressure. The experimentally observed time constants and peak pressure may be represented empirically:

$$\theta = 1/\lambda = 58\,(10^{-6})\, W^{1/3} \left(\frac{W^{1/3}}{R}\right)^{-0.22} \tag{2}$$

$$P_o = 2.16\,(10^{4}) \left(\frac{W^{1/3}}{R}\right)^{1.13} \tag{3}$$

where W = charge weight in lb.

R = radial distance from explosion in ft.

θ = time constant in sec.

P_o = peak pressure in lb./in.^{2}

The exponential shock wave of equation (1) may be represented by a Fourier integral:

$$p(t) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \phi(\omega)\, \epsilon^{i\omega t}\, d\omega \tag{4}$$

where

$$\varphi(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} p(\tau)\, \epsilon^{-i\omega\tau}\, d\tau \tag{5}$$

Combining (1) and (5), it may be shown that

$$\varphi(\omega) = P_0 / (2\pi)^{1/2}(\lambda + i\omega) \tag{6}$$

and that the absolute magnitude of φ is:

$$|\varphi(\omega)| = P_0 / (2\pi)^{1/2}(\lambda^2 + \omega^2)^{1/2} \tag{7}$$

2. Energy flux in the unattenuated wave.

The total energy flux in the shock wave is given by:

$$E = (\rho c)^{-1} \int_{-\infty}^{\infty} [p(t)]^2 \, dt \tag{8}$$

where ρc is the acoustic impedance of the water.

Using Plancherel's theorem:

$$E = (\rho c)^{-1} \int_{-\infty}^{\infty} |\varphi(\omega)|^2 \, d\omega \tag{9}$$

and combining equations (7) and (9):

$$E = \frac{P_0^2}{2\pi\rho c} \int_{-\infty}^{\infty} \frac{d\omega}{\lambda^2 + \omega^2} \tag{10}$$

Evaluating the integral in equation (10):

$$E = P_0^2 / 2\lambda\rho c \tag{11}$$

The distribution function of energy flux with respect to angular frequency is:

$$\mathcal{E}(\omega) = \frac{P_0^2}{2\pi\rho c} \cdot \frac{1}{\lambda^2 + \omega^2} \tag{12}$$

The normalized distribution function is thus given by:

$$\mathcal{E}_n(\omega) = \frac{\mathcal{E}}{E} = \frac{\lambda}{\pi} \cdot \frac{1}{\lambda^2+\omega^2} \tag{13}$$

The physical significance of equation (13) is

that $\int_{\omega_1}^{\omega_2} \mathcal{E}_n(\omega)\,d\omega$ represents the fraction of the total energy flux contributed

by angular frequencies in the range between ω_1 and ω_2. Similarly

$\int_{f_1}^{f_2} 2\pi\mathcal{E}_n(f)\,df$ represents the fraction of the total energy flux contributed

by frequencies in the range between f_1 and f_2.

3. Effect of viscous attenuation.

Following the method of reference (1) p. 38-59 and p. 74-76, we apply Liebermann's (2) frequency-attenuation results for sea water to the explosion pressure wave and introduce first order corrections for finite amplitude effects. The "equivalent viscous distance", χ, is given as a function of R in Fig. 10 of reference (1). Using Liebermann's data, the attenuation parameter, a, is given in terms of χ:

$$a = 7.11\,(10^{-8})\,\chi^{1/2} \tag{14}$$

It is then shown that the total energy flux, E^*, in the attenuated wave is given by:

$$E^* = \frac{P_0^2}{2\pi\rho c}\int_{-\infty}^{\infty} \frac{\epsilon^{-2a^2\omega^2}}{\lambda^2+\omega^2}\,d\omega \tag{15}$$

and evaluation of the integral in equation (15) gives:

$$E^* = \frac{P_0^2}{2\rho c} \cdot \frac{\epsilon^{2a^2\lambda^2}}{\lambda}\left[1 - Pi(2^{1/2}\lambda a)\right] \tag{16}$$

where $Pi(2^{1/2}\lambda a)$ is the conventional probability integral defined and tabulated in various Tables of Functions such as reference (3).

The energy distribution function is then:

$$\mathcal{E}^*(\omega) = \frac{P_0^2}{2\pi\rho c} \cdot \frac{\epsilon^{-2a^2\omega^2}}{\lambda^2+\omega^2} \tag{17}$$

and the normalised energy distribution function:

$$\mathcal{E}_n^*(\omega) = \frac{\lambda}{\pi(\lambda^2+\omega^2)} \cdot \frac{\epsilon^{-2a^2(\lambda^2+\omega^2)}}{[1 - Pi(2^{1/2}\lambda a)]} \tag{18}$$

By the same argument used above; $\int_{f_1}^{f_2} 2\pi\mathcal{E}^*(f)df$ represents the total energy flux in the frequency range between f_1 and f_2.

APPENDIX II

Tables of Energy Distribution Functions

W = Charge Weight = 0.5 lb.
R = Radial Distance from Charge, ft.
$\mathcal{E}^{a}(\omega)$ = Energy Flux Distribution Function

$$\mathcal{E}^{a}(\omega) = 4980\, P_o^2 \; \frac{\epsilon^{-2a^2\omega^2}}{\lambda^2 + \omega^2} \qquad \text{ergs/cm}^2$$

$$P_o = 2.16\,(10^4)\left(\frac{W^{1/3}}{R}\right)^{1.13} \qquad \text{lb/in}^2$$

Values of a^2 and λ^2 are given with each table.

TABLE 1

R = 1000 ft.; λ^2 = 20.5(10^6)sec^{-1}; a^2 = 7980(10^{-16}); P_o = 6.88 lb/in^2

Frequency f (kc)	$\mathcal{E}^{a}(\omega) \times 10^6$ ergs/cm^2	$\mathcal{E}^{a}(\omega)$ db above 10^{-6} ergs/cm^2
0.5	7760	38.9
1.0	3960	36.0
2.0	1320	31.2
5.0	233	23.7
10.0	59	17.7
15.0	26	14.2
20.0	14.5	11.6
25.0	9.2	9.6
30.0	6.2	7.9

TABLE 2

$R = 2500$ ft.; $\lambda^2 = 13.3(10^6)$; $a^2 = 34{,}800(10^{-16})$; $P_o = 2.44$ lb/in^2

Frequency f (kc)	$\mathcal{E}^*(\omega) \times 10^6$ ergs/cm^2	$\mathcal{E}^*(\omega)$ db above 10^{-6} ergs/cm^2
0.5	1290	31.1
1.0	563	27.5
2.0	173	22.4
5.0	29.6	14.7
10.0	7.32	8.6
15.0	3.13	5.0
20.0	1.69	2.3
25.0	1.01	0.00
30.0	0.65	-1.9

TABLE 3

$R = 5000$ ft.; $\lambda^2 = 10.1(10^6)$; $a^2 = 96{,}000(10^{-16})$; $P_o = 1.12$ lb/in^2

Frequency f (kc)	$\mathcal{E}^*(\omega) \times 10^6$ ergs/cm^2	$\mathcal{E}^*(\omega)$ db above 10^{-6} ergs/cm^2
0.5	313	25.0
1.0	126	21.0
2.0	36.9	15.7
5.0	6.14	7.9
10.0	1.46	1.6
15.0	0.588	-2.3
20.0	0.291	-5.4
25.0	0.157	-8.0
30.0	0.088	-10.6

TABLE 4

$R = 7500$ ft.; $\lambda^2 = 8.35(10^6)$; $a^2 = 167{,}000(10^{-16})$; $P_o = 0.70$ lb/in^2

Frequency f (kc)	$\mathcal{E}^*(\omega) \times 10^6$ ergs/cm^2	$\mathcal{E}^*(\omega)$ db above 10^{-6} ergs/cm^2
0.5	136	21.3
1.0	51.7	17.1
2.0	14.8	11.7
5.0	2.41	3.8
10.0	0.548	-2.6
15.0	0.206	-6.9
20.0	0.092	-10.4
25.0	0.0438	-13.6
30.0	0.0211	-16.8

TABLE 5

$R = 10{,}000$ ft.; $\lambda^2 = 7.40(10^6)$; $a^2 = 248{,}000(10^{16})$; $P_0 = 0.51$ lb/in^2

Frequency f (kc)	$\mathcal{E}^*(\omega) \times 10^6$ ergs/cm^2	$\mathcal{E}^*(\omega)$ db above 10^{-6} ergs/cm^2
0.5	75.6	18.8
1.0	27.7	14.4
2.0	7.82	8.9
5.0	1.25	1.0
10.0	0.271	-5.7
15.0	0.0938	-10.3
20.0	0.0375	-14.3
25.0	0.0154	-18.1
30.0	0.00625	-22.0

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3. B. O. Peirce. A Short Table of Integrals. Ginn and Co., p. 116-120.

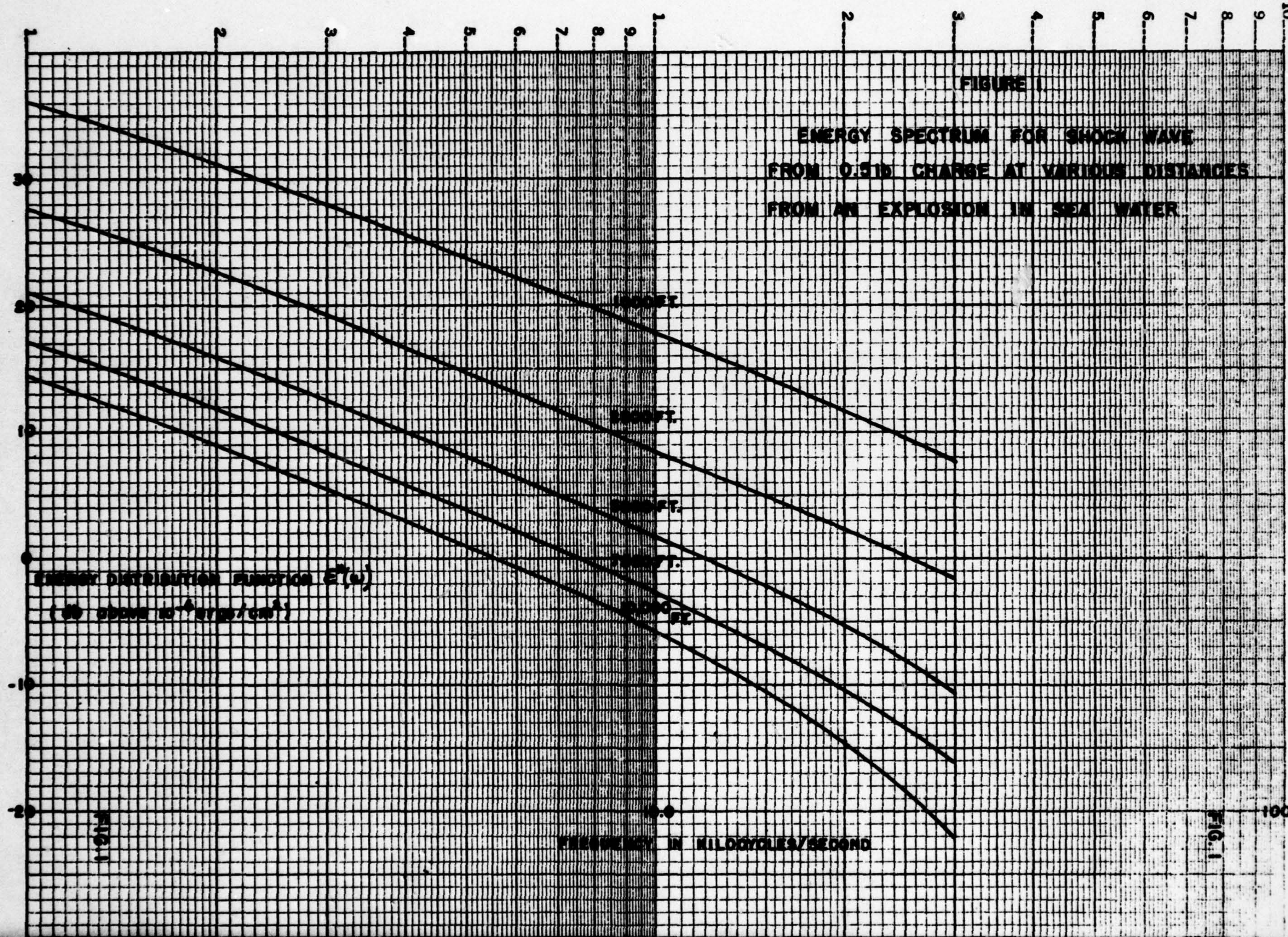
FIGURE I
ENERGY SPECTRUM FOR SHOCK WAVE
FROM 0.5lb CHARGE AT VARIOUS DISTANCES
FROM AN EXPLOSION IN SEA WATER
ENERGY DISTRIBUTION FUNCTION
FREQUENCY IN KILOCYCLES/SECOND
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0
-10
-20
10.0
100.0
FIG. I
FIG. I